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Effects of Temperature and Pressure on the Thermal Conductivities of Solids. Part I. The Effect of Temperature on the Thermal Conductivities of Some Electrical Insulators

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XII. *Effects of Temperature and Pressure on the Thermal Conductivities of Solids.—Part I. The Effect of Temperature on the Thermal Conductivities of some Electrical Insulators.*

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Introduction.

The question as to whether the thermal conductivity of a solid varies with temperature is an important one, and a considerable amount of attention has been bestowed on it. The experimental work which has been done cannot, however, be said to have led to a definite conclusion, owing to the discrepancies between the results obtained by different observers. Although the more recent of these results have produced a general disbelief of the idea which prevailed a few years ago, that the thermal conductivity of a solid would increase as the temperature increased, yet they do not justify the opposite conclusion being drawn.

Many of the discrepancies may no doubt be put down to the use of materials differing in purity or physical condition, although the effect of a small amount of impurity seems, from experiments which have been made on mixtures, to be much less in the case of thermal than it is in the case of electrical conductivity.

If the thermal conductivity does change with change of temperature, it is of importance that the differences of temperature used in the experimental work should be comparatively small, and that the mean temperature of the substance under test should be ascertained. Failure to comply with these requirements renders it unwise to take into account many of the older experiments in estimating the evidence for and against change of conductivity with temperature.

A further cause of disagreement may be found in the experimental methods used either not being amenable to mathematical treatment, or the treatment not having accurately expressed the experimental conditions.

Under these circumstances it seemed to me advisable to make a series of determinations of the thermal conductivities of a few representative substances under different pressures and at different temperatures, and the present paper gives an account of the first steps taken to this end.

It deals with the effect of temperature on the thermal conductivities of some electrical insulators which could readily be obtained in a pure state and were fairly representative substances.

The temperature interval used has been small, the actual temperature of the test has been observed, and the mathematical work has been made to follow the experimental conditions very closely.

Consideration of a Suitable Method.

A form of apparatus which lends itself readily to mathematical treatment is one in which the isothermal surfaces are concentric spheres, but the mechanical difficulties in the construction and use of such an apparatus render it unsuitable.

The mechanical difficulties disappear if cylindrical isothermal surfaces are substituted for spherical, and the mathematical treatment of such cases is comparatively simple if the cylinders are long enough to enable the effects of the ends to be neglected at points near the middle of the length.

If through an infinitely long, straight thin wire, embedded in an electrical insulator extending on all sides to infinity, an electric current is sent which, on account of the resistance of the wire, generates in each centimetre length of it an amount H of heat per second, the temperature v at a point whose perpendicular distance from the axis of the wire is r, is given by the equation $v = \mathrm{A} - \frac{\mathrm{H}}{2\pi k}\log r$, where A is a constant and k is the thermal conductivity of the medium surrounding the wire.

If the temperatures v_0 and v_1 are determined at two points r_0 and r_1, then

$$v_0 - v_1 = \frac{\mathrm{H}}{2\pi k}\log\frac{r_1}{r_0}.$$

Hence if v_0 and v_1 are observed, the thermal conductivity k of the medium surrounding the wire may be found.

It is, however, not possible to carry out the experiment in this simple form. The medium must in the first place be confined within a cylindrical surface of limited radius, and the effect of this boundary on the difference between v_0 and v_1 must be calculated.

For reasons which will be explained later, it is not advisable to have the axis of this cylinder coincident with that of the wire in which the heat is generated, but to have it midway between the points at which the temperatures are measured. The heating wire and two points at which temperatures are measured lie in a diametral plane of the cylinder, with the heating wire further from the axis than the two points at which temperatures are measured.

The conditions to which the surface of this cylinder is subjected influence the difference of temperature $v_0 - v_1$. If for example the cylinder be placed in a gas, it will lose heat from its surface at a rate which may, for a small range of temperature, be taken proportional to excess of the temperature of the surface over that of the gas, and at the surface we shall have

$$k \frac{\partial v}{\partial n} + h (v - V) = 0,$$

where k is the conductivity of the medium of the cylinder, n the normal to its surface, h the loss per second from 1 sq. centim. of surface when one degree hotter than the gas, V the temperature of the gas.

The expression for the temperature difference $v_0 - v_1$ then involves a term in h/k which, although not large, will vary with the material used and with the variations of h as the experiment is carried out at different temperatures.

In order to reduce as far as possible the effect of h on the temperature difference $v_0 - v_1$, the cylinder of material to be tested was cast in and kept surrounded by a brass cylinder with walls about ·5 millim. thick, which has the effect of making the surface of the material in contact with the brass nearly an isothermal surface. The value of h has then a negligible influence on the temperature difference observed, and its value need be known only approximately.

The measurement of the temperature difference $v_0 - v_1$ may be made in several ways, *e.g.*, by two thermometers with long thin bulbs arranged with their axes parallel to that of the wire, or by two similar bulbs of a small gas thermometer arranged to read differentially, or by two thermojunctions placed at the two points and connected in opposition, or by two straight wires of a metal of known coefficient of increase of electrical resistance with temperature, arranged parallel to the heating wire, and forming two sides of a resistance bridge.

Of these methods the latter seemed to promise best and was first tried. So long as the material in which the wires were embedded was not subjected to great changes of temperature, it gave excellent results, but when the apparatus was cooled down to the temperature of liquid air, the wires were found to change their resistance in an irregular manner, which was traced to the contraction and occasional cracking of the material during the cooling process.

Increasing the time of cooling through a given temperature interval did not get rid of this trouble, and after some time the wires were replaced by the two bulbs of a small differential air, and eventually hydrogen, thermometer.* This arrangement again answered extremely well at ordinary temperatures, but at the temperature of liquid air irregularities were met with which seemed due mainly to the capillary forces acting on the thread of strong sulphuric acid which acted as an index, being large compared to the forces due to the pressure of the hydrogen at these low temperatures.

* LEES, 'Proc. Manchester Lit. and Phil. Soc.,' xlvii., p. viii. (1902).

This method was in turn discarded, and it was eventually found that by reverting to the resistance method and using thin spirals of wire, instead of the straight wires used previously, the difficulties due to the contraction of the medium were overcome.

Description of Apparatus.

The apparatus in its final form (fig. 1) consisted of two vertical narrow glass tubes, A_1, A_2, 5·8 centims. long, ·17 centim. internal, ·25 centim. external diameter, to the upper ends of which were fused two wider vertical tubes, B_1, B_2, 22 centims. long, ·22 centim. internal, ·30 centim. external diameter. The tubes were fixed together so that the axes of the narrower tubes were parallel to each other, ·79 centim. apart.

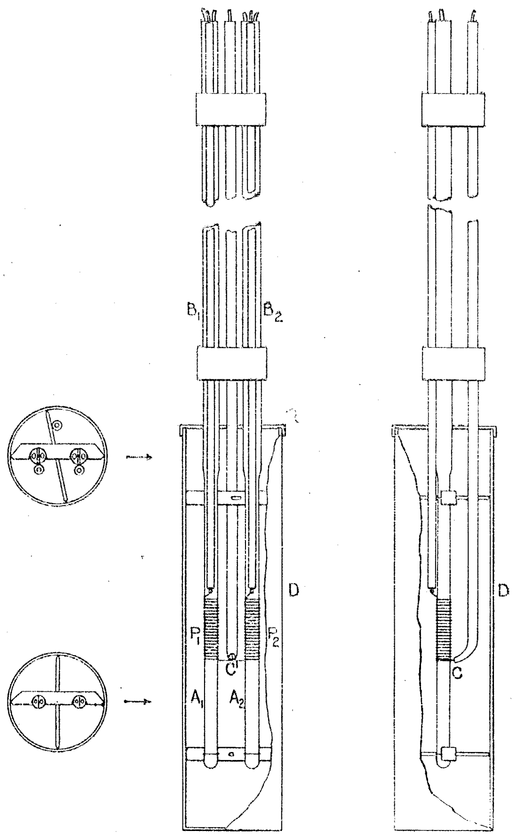

Fig. 1.

Down each of the wider tubes passed two bare No. 22 (·071 centim. diameter) copper wires, separated from each other by a thin glass tape extending along the centre of the tube. To the lower ends of the two copper wires in each tube were brazed the ends of a piece of No. 29 (·033 centim. diameter) platinoid wire, 11·2 centims. long, bent at its middle so that the two halves lay alongside each other in the narrower tube, insulated from each other by a narrower glass tape continuous with that in the wider tube. The space around the wires in the narrower tubes was filled with sealing-wax, and the lower ends of the tubes closed. In what follows, these tubes will be called the heating tubes.

Round the centre of each narrow tube was wound 14·5 centims. of No. 40 (·0122 centim. diameter) platinum wire, P_1, P_2, obtained from Messrs. JOHNSON and MATTEY as platinum thermometer wire. The coils of the spiral were at such a distance apart that the total length of spiral along each tube was 1·2 centims.

The lower ends of both spirals were soldered to the end C of a No. 20 (·091 centim. diameter) bare copper wire, which was surrounded by a thin glass tube of ·22 centim. outside diameter, fixed parallel to the heating tubes and ·6 centim. away from them. The lower end of this tube was bent towards the plane of the heating tubes, so that the end of the copper wire to which the spirals were attached came into the plane of those tubes.

The upper end of each spiral was soldered to a No. 20 bare copper wire enclosed in a thin glass tube in contact with the heating tubes. Each copper wire had a total length of 92 centims.

To give strength to the arrangement, the tubes passed through two short plugs of plaster of Paris, 10 and 22 centims. respectively from the lower ends of the heating tubes.

The material to be tested was fused in a brass tube D, 8·4 centims. long, 2·047 centims. external, 1·950 centims. internal diameter, with a base and lid of the same thickness.

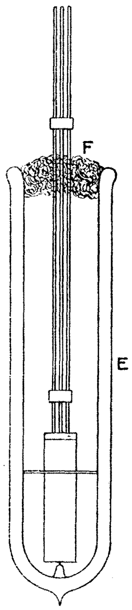

Fig. 2.

While the material was in the molten state, the testing apparatus $A_1A_2B_1B_2$ was slowly lowered into the tube till the platinoid wires of the heating coils occupied the middle two-thirds of the length of the brass tube. The heating tubes were kept in symmetrical positions laterally by two thin rods of wood of length equal to the internal diameter of the brass tube, which were attached to the tubes and projected equally from the two on each side. Through each wooden rod there passed at right angles a thin brass wire of length equal to the inside diameter of the brass tube.

The apparatus could thus be removed and replaced in the brass tube in the same symmetrical position.

The brass tube and its contents were placed near the bottom of a vertical straight Dewar tube E (fig. 2), 4·5 centims. internal diameter and 24 centims. internal length, and were held in place in the centre by three wires which extended horizontally from the brass tube to the surface of the Dewar tube.

The top of the Dewar tube was closed by a plug of cotton-wool, F (fig. 2), through which the conducting wires and their surrounding glass tubes passed to the cells and galvanometer.

The electric current for the heaters was supplied by three storage cells, and passed through regulating resistances (fig. 3) which could be adjusted by means of mercury-cup connections to ·01 ohm, and a Weston ammeter which had been carefully compared with a Kelvin current balance standardized by copper deposition. The potential difference at the ends of the copper leads attached to the heating coils was measured by a Keiser and Schmidt moving-coil voltmeter, standardized by the potentiometer method. A correction for the resistance of these leads was determined and applied to the readings thus obtained.

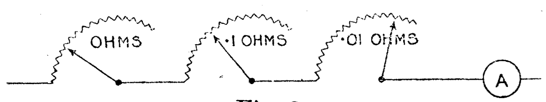

Fig. 3.

The bridge circuit for determining the resistance of one and the difference of resistance of the two platinum wire spirals was supplied with an independent current from a small Leclanché cell (fig. 4) with 53 ohms resistance in series with it.

The equal resistances, M_1M_2, which formed two sides of the bridge in the determination of the difference of resistance of the two spirals, each consisted of 50 centims. of No. 29 double silk-covered manganine wire, wound in parallel on a varnished block of wood, adjusted to equality within 1 part in 10,000, and varnished. The resistance of each coil, with its short copper leads, was found to be 4·850 B.A. ohms.

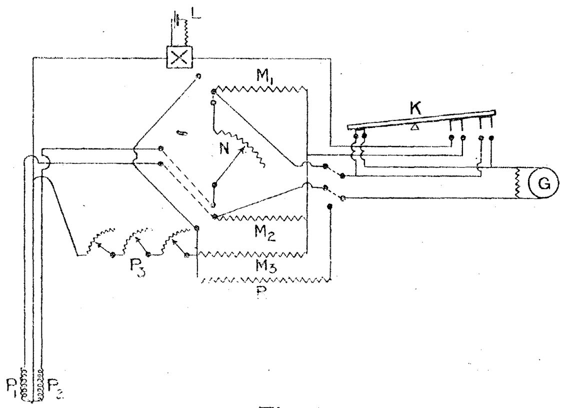

Fig. 4.

In series with that spiral P_1 or P_2, which in any experiment had the smaller resistance, could be placed an external resistance N capable of adjustment by means of mercury cups to ·01 ohm. This did not admit of an actual balance in the bridge, but the deflections of the galvanometer with the resistance adjusted a little above and a little below the value necessary for a balance were obtained, and the latter value calculated.

To fix the temperature at which each determination was made, the hotter spiral and the manganine coil in series with it could, by means of mercury-cup connections, be made into two of the resistances of a second bridge, the third arm of which was a coil of bare manganine wire, M_3, wound on a block of wood and varnished, and the fourth arm, P_3, a resistance box with ohms and tenths, and a mercury switch resistance capable of adjustment to ·01 ohm. These were so connected that the adjustable resistance was proportional to the resistance of the platinum spiral. The resistance of the coil in the third arm was for convenience taken such that the resistance of the fourth arm in tenth-ohms was approximately the temperature of the platinum spiral.

The galvanometer was of the moving coil type, had a resistance of 22 ohms, and was used with a shunt of 10 ohms across its terminals. It was connected to either bridge through a thermo-electric key, K, which carried three ∩-shaped conductors, the ends of which dipped into six mercury cups arranged as shown in fig. 4. When connected to the second bridge, a resistance R was placed in series with it to diminish its sensitiveness.

Method of Experimenting.

In making a test of a material, the material was melted in the brass tube D, the tube being for this purpose placed in a water or oil bath, the temperature of which was raised till it was just sufficient to melt the substance. The end A_1A_2 of the apparatus $A_1A_2B_1B_2$ which was to be placed in the material, was surrounded by a tube and placed in the bath to be heated to the same temperature. When the material in D was liquid, the apparatus was immersed to the requisite extent in it. The tube and contents were then removed from the bath, and cooled from underneath by immersion for about 1 centim. in cold water. As the liquid cooled and solidified in the lower part of the tube, more liquid was supplied at the top from a heated test tube. This was continued till the whole of the brass tube was filled with the solid material, to a little above the upper edge. The top was then heated and brought down on to the material, melting it as it came so that the two were in good thermal contact.

The apparatus was then placed centrally in the Dewar tube, as shown in fig. 2, the circuits made, and the difference of the resistance of the two platinum spirals and the resistance of one of them were then measured. If these were found to be normal, a current of 1 or 2 ampères was sent through one of the heating coils, and the two resistance measurements repeated after 5 and 10 minutes' intervals. If the two sets of observations were equal, or nearly so, the last observations were taken to represent the temperature for the steady state. During the intervals between the readings, any small change in the heating current was corrected by a re-adjustment of the external resistances. The heating current was then switched off the first heating coil and sent through the second, and the observations repeated. The mean of the two values of the difference of resistance of the platinum spiral, and the mean of the resistances of the hotter spiral in each case, were then taken. These means eliminate from the result any want of symmetry in the heating and temperature measuring coils, and in their positions in the brass tube.

If any unusual difference between the observations with the heating currents through the two coils was observed, a third set of observations with the heating current again through the first coil was taken, and the mean result for these two observations was combined with the result with the heating current through the other coil.

These tests having shown that the apparatus was in working order, it was removed from the Dewar tube, which was then half filled with liquid air, and the apparatus lowered very slowly into it, so that at first not more than 1 centim. of the brass tube would be immersed, till a considerable length of the tube and contents had been cooled down to a low temperature. Then the tube would be lowered still further, and so on till completely immersed.

When this was the case, a thick plug of cotton wool was placed in the top of the Dewar tube.

During the cooling process an occasional test of resistance would be made to see how the cooling was proceeding.

When the tube was completely immersed in the liquid air, the heating current was adjusted to 1 or 2 ampères, as the case might be, and the observations taken as described above.

When sufficient observations at the temperature of the liquid air had been made, the Dewar tube was tilted in such a way that the liquid air would run out without the apparatus being disturbed.

The temperature of the apparatus then rose, and during the rise observations were taken at frequent intervals with the heating current alternately through one and the other heating coil. The current used in the bridge circuit was occasionally reversed in order to see that no serious thermal electromotive forces were present in the circuit.

Observations were thus obtained up to about the temperature the material had at the commencement, and the agreement of the observations with those previously taken was a test of the constancy of the conditions under which the observations were made.

Theory of the Apparatus.

The complete theory of a finite straight-line source of heat within and parallel to the axis of a finite cylinder of conducting material, surrounded by a concentric shell of a second conducting material whose outer surfaces are exposed to a gas, requires the evaluation of a number of definite integrals involving Bessel functions, and has not been worked out.

In the apparatus used, however, the concentric shell surrounding the cylinder is of metal whose thermal conductivity is 200 or 300 times that of the cylinder. The shell may, therefore, with a close degree of approximation, be taken as an isothermal surface.

Let a cylinder of conducting material, of thermal conductivity k, of length $2l$, and radius a, have within it a steady straight-line source of heat parallel to the axis of the cylinder, and distant c from it, and let its external surfaces be maintained at a constant temperature. To find the distribution of temperature throughout the cylinder, given that of the source.

If v is the temperature at any point of the cylinder distant x from the mid-cross-section and r from the axis in an axial plane inclined at θ to that of reference, V the temperature of its surface, the following equations must be satisfied

$$\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial r^2}+\frac{1}{r}\frac{\partial v}{\partial r}+\frac{1}{r^2}\frac{\partial^2 v}{\partial \theta^2} = 0 \text{ throughout the cylinder} \quad (1).$$

$$v = \mathrm{V} \quad \text{at} \quad r = a \quad (2),$$

$$v = \mathrm{V} \quad \text{at} \quad x = \pm l \quad (3).$$

If the source is symmetrical with respect to the mid-cross-section of the cylinder, and its intensity is zero at $x = \pm l$, it may be expressed by a Fourier series whose general term is $\mathrm{H}_n \cos n\pi \dfrac{x}{2l}$, where n is an odd integer.

A straight-line source of strength $\mathrm{H}_n \cos \dfrac{n\pi x}{2l}$ in an infinite solid of thermal conductivity k produces at the point $x\rho$ a temperature

$$v_n = \frac{\mathrm{H}_n}{2\pi k} \cos \frac{n\pi x}{2l} \cdot \mathrm{K}_0\left(\frac{n\pi\rho}{2l}\right),$$

where K is the function to which BESSEL'S function of the second kind reduces for a pure imaginary value of the argument.

Since the limit of $2\pi\rho k \frac{\partial v_n}{\partial \rho}$, when $\rho = 0$, is $\rho H_n \cos\frac{n\pi x}{2l} \cdot \frac{1}{\rho}$, *i.e.*, $H_n \cos\frac{n\pi x}{2l}$, we see that the expression for v_n satisfies the necessary conditions.

If the origin of the r co-ordinates be taken at a point distant c from that of the ρ co-ordinates, $\rho = \sqrt{c^2+r^2-2cr\cos\theta}$, where θ is the angle which the r co-ordinate makes with the line through the source.

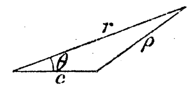

Fig. 5.

Hence, in terms of r, c, and x, the temperature due to the source $H_n \cos\frac{n\pi x}{2l}$, placed at a distance c from the new axis, is

$$v_n = \frac{H_n}{2\pi k}\cos\frac{n\pi x}{2l} \cdot K_0\left(\frac{n\pi}{2l}\sqrt{c^2+r^2-2cr\cos\theta}\right).$$

To expand the K_0 function in a Fourier series in terms of θ, we have, if Y_0 is the Bessel function of the second kind and zero order, and $r>c$,

$$Y_0(\alpha\sqrt{c^2+r^2-2cr\cos\theta}) = J_0(\alpha c)\,.\,Y_0(\alpha r)+2\sum_{\beta=1}^{\infty} J_\beta(\alpha c)Y_\beta(\alpha r)\cos\beta\theta$$

(HEINE, 'Kugelfunctionen,' I., p. 342). Therefore

$$Y_0(i\alpha\sqrt{c^2+r^2-2cr\cos\theta}) = J_0(i\alpha c)Y_0(i\alpha r)+2\sum_{\beta=1}^{\infty} J_\beta(i\alpha c)Y_\beta(i\alpha r)\cos\beta\theta,$$

where $i = \sqrt{-1}$.

But

$$i^\beta I_\beta(\alpha c) = J_\beta(i\alpha c)$$

(GRAY and MATHEWS, 'Bessel Functions,' p. 66), and

$$i^\beta K_\beta(\alpha r) = Y_\beta(i\alpha r).$$

Therefore

$$K_0(\alpha\sqrt{c^2+r^2-2cr\cos\theta}) = I_0(\alpha c)K_0(\alpha r)+2\sum_{\beta=1}^{\infty}(-1)^\beta I_\beta(\alpha c)K_\beta(\alpha r)\cos\beta\theta,$$

for $r>c$.

Hence the temperature v_n in an infinite cylinder due to a source $H_n\cos\frac{n\pi x}{2l}$ distant c from the axis is, for points at a distance from the axis greater than c, given by

$$v_n = \frac{H_n}{2\pi k}\cos\frac{n\pi x}{2l}\left\{I_0\left(\frac{n\pi c}{2l}\right)K_0\left(\frac{n\pi r}{2l}\right)+2\sum_{\beta=1}^{\infty}(-1)^\beta I_\beta\left(\frac{n\pi c}{2l}\right)K_\beta\left(\frac{n\pi r}{2l}\right)\cos\beta\theta\right\}.$$

For points nearer to the axis than the source, r and c must be interchanged.

Since $\cos\alpha x I_\beta(\alpha r)\,.\,\cos\beta\theta$ is also a solution of equation (1), we may add to v_n terms of the form $\cos\alpha x I_\beta(\alpha r)\cos\beta\theta$; which, since I_β has no singular points, do not change the strength of the source $H_n\cos\frac{n\pi x}{2l}$.

Hence we have the more general solution

$$v_n = \frac{H_n}{2\pi k}\cos\frac{n\pi x}{2l}\left[I_0\left(\frac{n\pi c}{2l}\right)\left\{K_0\left(\frac{n\pi r}{2l}\right)+A_0 I_0\left(\frac{n\pi r}{2l}\right)\right\} + 2\sum_{\beta=1}^{\infty}(-1)^\beta I_\beta\left(\frac{n\pi c}{2l}\right)\left\{K_\beta\left(\frac{n\pi r}{2l}\right)+A_\beta I_\beta\left(\frac{n\pi r}{2l}\right)\right\}\cos\beta\theta\right],$$

where $A_0 \ldots A_\beta$ are arbitrary constants.

But at $r = a$, $v_n = V$ for all values of x.

Therefore

$$A_\beta = -K_\beta\left(\frac{n\pi a}{2l}\right)\Big/ I_\beta\left(\frac{n\pi a}{2l}\right)$$

and

$$v_n - V = \frac{H_n}{2\pi k}\cos\frac{n\pi x}{2l}\left[I_0\left(\frac{n\pi c}{2l}\right)\left\{K_0\left(\frac{n\pi r}{2l}\right)-\frac{K_0\left(\frac{n\pi a}{2l}\right)}{I_0\left(\frac{n\pi a}{2l}\right)} I_0\left(\frac{n\pi r}{2l}\right)\right\} + 2\sum_{\beta=1}^{\infty}(-1)^\beta I_\beta\left(\frac{n\pi c}{2l}\right)\left\{K_\beta\left(\frac{n\pi r}{2l}\right)-\frac{K_\beta\left(\frac{n\pi a}{2l}\right)}{I_\beta\left(\frac{n\pi a}{2l}\right)} I_\beta\left(\frac{n\pi r}{2l}\right)\right\}\cos\beta\theta\right],$$

which satisfies equations (1), (2), and (3), and corresponds to a source of strength $H_n \cos\frac{n\pi x}{2l}$ situated c from the axis of the cylinder.

If the source is of intensity $= \sum_n H_n \cos\frac{n\pi x}{2l}$, the temperature $= v = \sum_n v_n - V$.

In the apparatus used the source had a constant intensity H from $x = -\frac{2}{3}l$ to $x = +\frac{2}{3}l$, and the temperature was $= V$ at $x = \pm l$.

The intensity may therefore be represented by the Fourier series

$$\sum_n H_n \cos\frac{n\pi x}{2l}, \quad \text{where} \quad H_n = \frac{1}{l}\int_0^{2l} H\cos\frac{n\pi t}{2l}\,.\,dt$$

$$= \frac{1}{l}\int_0^{\frac{2}{3}l} H\cos\frac{n\pi t}{2l}\,.\,dt - \frac{1}{l}\int_{\frac{4}{3}l}^{2l} H\cos\frac{n\pi t}{2l}\,.\,dt$$

$$= \frac{H}{l}\cdot\frac{2l}{n\pi}\left\{\Big/\sin\frac{n\pi t}{2l}\Big/_0^{\frac{2}{3}l} - \Big/\sin\frac{n\pi t}{2l}\Big/_{\frac{4}{3}l}^{2l}\right\}$$

$$= \frac{2H}{n\pi}\left\{\sin\frac{n\pi}{3}+\sin\frac{2n\pi}{3}\right\}$$

$$= \frac{2H}{n\pi}\{1-(-1)^n\}\sin\frac{n\pi}{3}\,.$$

Therefore

$$v-\mathrm{V} = \frac{\mathrm{H}}{2\pi k}\cdot\frac{2}{\pi}\cdot\sum_{n=1}^{\infty}\frac{1}{n}\{1-(-1)^n\}\sin\frac{n\pi}{3}\cos\frac{n\pi x}{2l}$$

$$\left[\mathrm{I}_0\left(\frac{n\pi c}{2l}\right)\left\{\mathrm{K}_0\left(\frac{n\pi r}{2l}\right)-\frac{\mathrm{K}_0\left(\frac{n\pi a}{2l}\right)}{\mathrm{I}_0\left(\frac{n\pi a}{2l}\right)}\cdot\mathrm{I}_0\left(\frac{n\pi r}{2l}\right)\right\}\right.$$
$$\left.+2\sum_{\beta=1}^{\infty}(-1)^\beta\mathrm{I}_\beta\left(\frac{n\pi c}{2l}\right)\left\{\mathrm{K}_\beta\left(\frac{n\pi r}{2l}\right)-\frac{\mathrm{K}_\beta\left(\frac{n\pi a}{2l}\right)}{\mathrm{I}_\beta\left(\frac{n\pi a}{2l}\right)}\mathrm{I}_\beta\left(\frac{n\pi r}{2l}\right)\right\}\cos\beta\theta\right]$$

$$= \frac{\mathrm{H}}{2\pi k}\cdot\frac{2}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\{1-(-1)^n\}\sin\frac{n\pi}{3}\cos\frac{n\pi x}{2l}$$

$$\left[\mathrm{K}_0\left(\frac{n\pi\rho}{2l}\right)-\mathrm{I}_0\left(\frac{n\pi c}{2l}\right)\cdot\frac{\mathrm{K}_0\left(\frac{n\pi a}{2l}\right)}{\mathrm{I}_0\left(\frac{n\pi a}{2l}\right)}\mathrm{I}_0\left(\frac{n\pi r}{2l}\right)\right.$$
$$\left.-2\sum_{\beta=1}^{\infty}(-1)^\beta\mathrm{I}_\beta\left(\frac{n\pi c}{2l}\right)\frac{\mathrm{K}_\beta\left(\frac{n\pi a}{2l}\right)}{\mathrm{I}_\beta\left(\frac{n\pi a}{2l}\right)}\cdot\mathrm{I}_\beta\left(\frac{n\pi r}{2l}\right)\cos\beta\theta\right],$$

where $\rho = \sqrt{c^2+r^2-2cr\cos\theta}$.

When n is large, the term

$$(-1)^\beta\mathrm{I}_\beta\left(\frac{n\pi c}{2l}\right)\frac{\mathrm{K}_\beta\left(\frac{n\pi a}{2l}\right)}{\mathrm{I}_\beta\left(\frac{n\pi a}{2l}\right)}\mathrm{I}_\beta\left(\frac{n\pi r}{2l}\right)$$

becomes equal to

$$\frac{l}{n\pi\sqrt{cr}}e^{-\frac{n\pi}{2l}(2a-c-r)}$$

(GRAY and MATHEWS, p. 68). The series in square brackets is therefore convergent.

If the term within square brackets be written u, u satisfies the equation

$$\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}+\frac{1}{r^2}\frac{\partial^2 u}{\partial\theta^2}-\left(\frac{n\pi}{2l}\right)^2 u = 0.$$

The mean value of u taken along any circle of radius p in a plane perpendicular to the x axis and enclosing no singularities is therefore equal to $\mathrm{I}_0\left(\frac{n\pi p}{2l}\right)\cdot u_0$, where u_0 is the value of u at the centre of the circle.* Hence the mean value of u taken over

* H. WEBER, 'Math. Ann.,' vol. 1, p. 9 (1869).

the surface of a cylinder of radius r, with its axis parallel to the x axis, is equal to $I_0\left(\frac{n\pi p}{2l}\right)$ times the mean value of u along the axis of the cylinder.

If the circle of radius p has its centre q from the singularity $f(x) \, . \, K_0\left(\frac{n\pi\rho}{2l}\right)$, and encloses that singularity, we have, since

$$\rho = \sqrt{q^2+p^2+2qp\cos\theta} \quad \text{and} \quad p > q,$$

$$K_0\left(\frac{n\pi\rho}{2l}\right) = I_0\left(\frac{n\pi q}{2l}\right) K_0\left(\frac{n\pi p}{2l}\right) + 2\Sigma\,(-1)^\beta I_\beta\left(\frac{n\pi q}{2l}\right) K_\beta\left(\frac{n\pi p}{2l}\right)\cos\beta\theta.$$

Hence the mean value of $K_0\left(\frac{n\pi\rho}{2l}\right)$ along the circle

$$= I_0\left(\frac{n\pi q}{2l}\right) K_0\left(\frac{n\pi p}{2l}\right),$$

and the mean value over the surface of a circular cylinder of radius p with its axis parallel to the source and distance q from it $= I_0\left(\frac{n\pi q}{2l}\right) K_0\left(\frac{n\pi p}{2l}\right)$ times the mean value of $f(x)$ along the length of the source within the cylinder.

The two spirals of thin platinum wire, whose resistances give their mean temperatures, had the same radius p, were of the same length, $2s$, and were situated at the same distance from t e axis of the cylinder. One had its axis coincident with the source. The temperature $\bar{v}_1$ indicated by it was therefore $= K_0\left(\frac{n\pi p}{2l}\right)$ times the mean value of $f(x)$ along the length $2s$ of the source within it, $+ I_0\left(\frac{n\pi p}{2l}\right)$ times the value of v due to the remainder of the expression for v above.

I.e.,

$$\bar{v}_1 - V = \frac{H}{2\pi k}\frac{2}{\pi}\sum_{n=1}\frac{1}{n}\{1-(-1)^n\}\sin\frac{n\pi}{3}\,.\,\frac{1}{s}\int_0^s\cos\frac{n\pi x}{2l}\,.\,dx$$

$$\left[\begin{array}{l} K_0\left(\frac{n\pi p}{2l}\right) \\ -I_0\left(\frac{n\pi p}{2l}\right)\left\{I_0\left(\frac{n\pi c}{2l}\right)\right\}^2 \dfrac{K_0\left(\frac{n\pi a}{2l}\right)}{I_0\left(\frac{n\pi a}{2l}\right)} \\ -2I_0\left(\frac{n\pi p}{2l}\right)\sum_{\beta=1}^{\infty}(-1)^\beta\left\{I_\beta\left(\frac{n\pi c}{2l}\right)\right\}^2\dfrac{K_\beta\left(\frac{n\pi a}{2l}\right)}{I_\beta\left(\frac{n\pi a}{2l}\right)}\cos\beta\theta \end{array}\right].$$

Or, since for this spiral $\theta = 0$,

$$\bar{v}_1 - \mathrm{V} = \frac{\mathrm{H}}{2\pi k} \cdot \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{1}{n} \{1-(-1)^n\} \sin\frac{n\pi}{3} \cdot \frac{\sin\frac{n\pi s}{2l}}{\frac{n\pi s}{2l}} \left[\begin{array}{l} \mathrm{K}_0\left(\frac{n\pi p}{2l}\right) - \mathrm{I}_0\left(\frac{n\pi p}{2l}\right)\left\{\mathrm{I}_0\left(\frac{n\pi c}{2l}\right)\right\}^2 \frac{\mathrm{K}_0\left(\frac{n\pi a}{2l}\right)}{\mathrm{I}_0\left(\frac{n\pi a}{2l}\right)} \\ -2\mathrm{I}_0\left(\frac{n\pi p}{2l}\right) \sum_{\beta=1}^{\infty} (-1)^\beta \left\{\mathrm{I}_\beta\left(\frac{n\pi c}{2l}\right)\right\}^2 \frac{\mathrm{K}_\beta\left(\frac{n\pi a}{2l}\right)}{\mathrm{I}_\beta\left(\frac{n\pi a}{2l}\right)} \end{array} \right].$$

Similarly for the mean temperature $\bar{v}_2$ of the second spiral, for which $\rho = 2c$, $\theta = \pi$, we have

$$\bar{v}_2 - \mathrm{V} = \frac{\mathrm{H}}{2\pi k} \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{1}{n} \{1-(-1)^n\} \sin\frac{n\pi}{3} \cdot \frac{\sin\frac{n\pi s}{2l}}{\frac{n\pi s}{2l}} \mathrm{I}_0\left(\frac{n\pi p}{2l}\right)$$

$$\left[\begin{array}{l} \mathrm{K}_0\left(\frac{n\pi 2c}{2l}\right) - \left\{\mathrm{I}_0\left(\frac{n\pi c}{2l}\right)\right\}^2 \frac{\mathrm{K}_0\left(\frac{n\pi a}{2l}\right)}{\mathrm{I}_0\left(\frac{n\pi a}{2l}\right)} \\ -2 \sum_{\beta=1}^{\infty} (-1)^\beta \left\{\mathrm{I}_\beta\left(\frac{n\pi c}{2l}\right)\right\}^2 \frac{\mathrm{K}_\beta\left(\frac{n\pi a}{2l}\right)}{\mathrm{I}_\beta\left(\frac{n\pi a}{2l}\right)} (-1)^\beta \end{array} \right].$$

Hence, for the difference $\bar{v}_1 - \bar{v}_2$ of mean temperature of the spirals, we have

$$\bar{v}_1 - \bar{v}_2 = \frac{\mathrm{H}}{2\pi k} \cdot \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{1}{n} \{1-(-1)^n\} \sin\frac{n\pi}{3} \frac{\sin\frac{n\pi s}{2l}}{\frac{n\pi s}{2l}} \left[\begin{array}{l} \mathrm{K}_0\left(\frac{n\pi p}{2l}\right) - \mathrm{I}_0\left(\frac{n\pi p}{2l}\right) \mathrm{K}_0\left(\frac{n\pi c}{l}\right) \\ +4\mathrm{I}_0\left(\frac{n\pi p}{2l}\right) \sum_{\beta=1,3,5}^{\infty} \left\{\mathrm{I}_\beta\left(\frac{n\pi c}{2l}\right)\right\}^2 \frac{\mathrm{K}_\beta\left(\frac{n\pi a}{2l}\right)}{\mathrm{I}_\beta\left(\frac{n\pi a}{2l}\right)} \end{array} \right],$$

or,

$$\bar{v}_1 - \bar{v}_2 = \frac{\mathrm{H}}{2\pi k} \frac{4l}{\pi^2 s} \sum_{n=1}^{\infty} \frac{1}{n^2} \{1-(-1)^n\} \sin\frac{n\pi}{3} \sin\frac{n\pi s}{2l} \left[\begin{array}{l} \mathrm{K}_0\left(\frac{n\pi p}{2l}\right) - \mathrm{I}_0\left(\frac{n\pi p}{2l}\right) \mathrm{K}_0\left(\frac{n\pi c}{l}\right) \\ +4\mathrm{I}_0\left(\frac{n\pi p}{2l}\right) \sum_{\beta=1,3,5}^{\infty} \left\{\mathrm{I}_\beta\left(\frac{n\pi c}{2l}\right)\right\}^2 \frac{\mathrm{K}_\beta\left(\frac{n\pi a}{2l}\right)}{\mathrm{I}_\beta\left(\frac{n\pi a}{2l}\right)} \end{array} \right].$$

In the apparatus used $l = 4{\cdot}2$ centims., $s = {\cdot}6$ centim. Therefore $\frac{s}{l} = \frac{1}{7}$.

$$p = {\cdot}131. \quad \text{Therefore } \frac{\pi p}{2l} = {\cdot}0490.$$

$$c = {\cdot}395. \quad ,, \quad \frac{\pi c}{2l} = {\cdot}1477. \quad \frac{\pi c}{l} = {\cdot}2954.$$

$$a = {\cdot}975. \quad ,, \quad \frac{\pi a}{2l} = {\cdot}3646.$$

Therefore

$$\bar{v}_1 - \bar{v}_2 = \frac{H}{2\pi k} \cdot \frac{28}{\pi^2} \sum_{n=1}^{\infty} \frac{1}{n^2} \{1-(-1)^n\} \sin \frac{n\pi}{3} \sin \frac{n\pi}{14}$$
$$\left[K_0({\cdot}049n) + I_0({\cdot}049n) \left[-K_0({\cdot}295n) + 4 \sum_{\beta=1,3,5}^{\infty} \{I_\beta({\cdot}148n)\}^2 \frac{K_\beta({\cdot}365n)}{I_\beta({\cdot}365n)} \right] \right].$$

Since terms of the form

$$\cos \alpha x \left[A_0 I_0(\alpha r) + \sum_{\beta=2,4,6}^{\infty} A_\beta I_\beta(\alpha r) \cos \beta\theta \right]$$

have disappeared from the final equation for $\bar{v}_1 - \bar{v}_2$, the same value of the temperature difference would be obtained from assuming that the brass tube was not at a uniform temperature V, but at a temperature

$$V + \cos \alpha x \left[A_0 I_0(\alpha r) + \sum_{\beta=2,4,6}^{\infty} A_\beta I_\beta(\alpha r) \cos \beta\theta \right],$$

where α, A_β are arbitrary constants.

If, further, the temperature of the brass tube is a function of the time, symmetrical about the axis of the tube, we add to the preceding solution terms of the form

$$\cos \alpha x I_0(\gamma r) \, . \, e^{\delta t},$$

without altering the value of the observed temperature difference. This furnishes the justification for the use of the apparatus while its temperature is increasing or decreasing owing to its relation to its surroundings.*

The above theory has been worked out on the assumption that the heating wires and temperature spirals were directly embedded in the material of conductivity k. In the apparatus used the heating wires were enclosed in narrow glass tubes round

* It is however still possible that if the rate of change of temperature is great, the change of conductivity or specific heat of the substance may be sufficiently large to affect the result. In no case has any effect of the rate of heating on the result been detected.

which the temperature spirals were wound, and it is necessary to determine whether this will have an appreciable effect on the result.

For this purpose we may assume the cylinder of material to be infinitely long.

The source at C, fig. 6, is surrounded by a narrow glass cylinder, whose effect is merely to decrease or increase the temperature of all points external to it by a constant amount depending on the thermal conductivities of the glass and the material surrounding it. It produces therefore no change in the difference of temperature between two points in the material tested. If the narrow glass tube round C′ were absent, the temperature at a point P within the material would be that due to the source at C, and since the circle AA′ is at a constant temperature, an equal sink at C_1 the image of C_1, where $OC_1 = OA^2/OC$. We may therefore put it $= \frac{q}{2\pi k} \log \frac{C_1P}{CP}$, where q is the strength of the source and k the thermal conductivity of the medium. The presence of the glass tube at C′ makes it necessary to add to this expression terms representing a source and an equal sink whose strengths depend on the relative conductivities of the material and glass, and whose positions are C′ and C'_1, the images of C and C_1 respectively, with respect to the circle representing the section of the glass. The complete expression for the temperature at P will therefore be $\frac{q}{2\pi k} \log \frac{C_1P}{CP} + \frac{q'}{2\pi k} \log \frac{C'_1P}{C'P}$. But the mean value of the temperature along the circumference of a circle of radius p enclosing a source of strength q' is equal to $\frac{q'}{2\pi k} \log . \frac{1}{p}$, where k is the conductivity of the medium. Hence for any circle enclosing both source and sink at C′ and C'_1 it is zero. Since the temperature is determined by the resistance of a coil of wire wound round the glass tube, the coil encloses both C′ and C'_1 and the tube has no effect on the observed temperature difference between that coil and the one wound round the source.

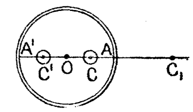

Fig. 6.

The following table shows the values of the functions which enter into the expression for k in terms of the difference of temperature observed :—

	$n = 1.$	
$\cdot 148n =$	·148	
$\cdot 365n =$	·365	
$\cdot 295n =$	·295	
$\cdot 049n =$	·049	
	$\beta = 1.$	$\beta = 3.$
$I_\beta(\cdot 148n)$	·0742	·0000675
$\{I_\beta(\cdot 148n)\}^2$	·00551	$\cdot 0^{8}.456$
$K_\beta(\cdot 365n)$	– 2·432	– 162·1
$I_\beta(\cdot 365n)$	·1856	·00102
$\{I_\beta(\cdot 148n)\}^2 \dfrac{K_\beta(\cdot 365n)}{I_\beta(\cdot 365n)}$	– ·0722	– $\cdot 0^{3}.725$
$4\Sigma\{I_\beta(\cdot 148n)\}^2 \dfrac{K_\beta(\cdot 365n)}{I_\beta(\cdot 365n)}$	– ·2917	
$K_0(\cdot 295n)$	1·384	
$-K_0(\cdot 295n) + 4\Sigma\{I_\beta(\cdot 148n)\}^2 \dfrac{K_\beta(\cdot 365n)}{I_\beta(\cdot 365n)}$	– 1·676	
$I_0(\cdot 049n)$	1·0006	
$I_0(\cdot 049n)\left[-K_0(\cdot 295n) + 4\Sigma\{I_\beta(\cdot 148n)\}^2 \dfrac{K_\beta(\cdot 365n)}{I_\beta(\cdot 365n)}\right]$	– 1·677	
$K_0(\cdot 049n)$	3·134	
$\left[K_0(\cdot 049n) + I_0(\cdot 049n)\left\{-K_0(\cdot 295n) + 4\Sigma\{I_\beta(\cdot 148n)\}^2 \dfrac{K_\beta(\cdot 365n)}{I_\beta(\cdot 365n)}\right\}\right]$	1·457	
n^2	1	
$\{1-(-1)^n\} \sin \dfrac{n\pi}{3}$	1·732	
$\sin \dfrac{n\pi}{14}$	2224	
$\dfrac{1}{n^2}\{1-(-1)^n\} \sin \dfrac{n\pi}{3} \sin \dfrac{n\pi}{14}$	·3853	
$\dfrac{1}{n^2}\{1-(-1)^n\} \sin \dfrac{n\pi}{3} \sin \dfrac{n\pi}{14}\left[\ldots\right]$	·5613	
$\Sigma \dfrac{1}{n^2}\{1-(-1)^n\} \sin \dfrac{n\pi}{3} \sin \dfrac{n\pi}{14}\left[\ldots\right]$	—	
$\dfrac{28}{\pi^2} \Sigma \dfrac{1}{n^2}\{1-(-1)^n\} \sin \dfrac{n\pi}{3} \sin \dfrac{n\pi}{14}\left[\ldots\right]$	—	

It may be noted for comparison that if the cylinder were assumed infinite in length the value expression would be

PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

	$n=5$.		$n=7$.		$n=11$.	$n=13$.	
	·740 1·825 1·475 ·245		1·036 2·555 2·065 ·343		1·62 4·02 3·25 ·539	1·92 4·74 3·84 ·637	
	$\beta=1$.	$\beta=3$.	$\beta=1$.	$\beta=3$.	$\beta=1$.	$\beta=1$.	
	·396	·00878	·591	·0246	1·106	1·476	
	·1568	·0⁴.771	·349	·0³.605	1·223	2·179	
	− ·176	− ·907	− ·0693	− ·246	− ·0122	− ·0055	
	− 1·349	·157	2·645	·522	9·938	19·17	
	− ·0204	− ·000445	− ·00915	− ·000285	− ·00150	− ·000626	
	− ·0834		− ·0367		− ·0060	− ·0025	
	·2209		·1062		·0260	·0134	
	− ·304		− ·143		− ·0320	− ·0159	
	1·015		1·030		1·074	1·104	
	− ·309		− ·147		− ·0344	− ·0176	
	1·567		1·254		·864	·732	
	1·258		1·107		·830	·714	
	25		49		121	169	
	− 1·732		1·732		− 1·732	1·732	
	·9010		1·00		·6234	·2224	
	− ·0624		·0353		− ·0089	·0023	
	− ·0785		·0391		− ·0074	·0016	
	—		—		—	—	·5161
	—		—		—	—	1·459

of this quantity would be $\log_e \dfrac{C_1C\,.\,C'C}{p\,.\,C'C_1} = 1\cdot466$ (fig. 6), and for many purposes this simple sufficiently accurate.

Standardisation of the Measurements.

Temperature Measurements.—The No. 40 platinum wire of the spirals used in the temperature measurements was obtained from Messrs. JOHNSON and MALLY specially for platinum thermometer work. In order to determine whether the resistance of the spirals was normal they were tested twice during the course of the work in steam ice, and liquid air, and the "fundamental constants" of CALLENDAR* calculated.

As stated on pp. 436, 437, one end of each spiral was soldered to 92 centims. of No. 20 copper wire, and both the other ends to a third copper wire of the same length and diameter. The glass tubes carrying the coils were inserted in an ordinary boiling-point apparatus as used for thermometers, and the copper leads brought to the terminals of a standard resistance box. The connections could be made to the ends AB, AC, or CB (see fig. 7). If R_{AB}, R_{AC}, and R_{CB} are the three resistances observed, the resistances of the coils themselves are $R_{AB}-R_{CB}$ and $R_{AB}-R_{AC}$ respectively. The apparatus was then placed in crushed ice and finally in liquid air contained in a Dewar tube and the measurements repeated. The temperature of the liquid air was read on a petroleum ether thermometer which had been standardised at the Reichsanstalt. One of the two tests which gave almost identical results is given below.

Fig. 7.

Determination of "fundamental constants" of platinum coils 03.12.9.

Resistances determined on standard box with ratio 1000 : 10 throughout.

Barometer, corrected 73·60 centims., therefore boiling-point = 99°·10 C.

In steam at this temperature,

$$R_{AC} = 1·9170, \quad R_{CB} = 1·9061, \quad R_{AB} = 3·7657 \text{ ohms.}$$

Hence coil in AC = 1·8596, coil in CB = 1·8487 ohms at 99°·1 C. = 372°·1 abs.

In ice at 0° C.,

$$R_{AC} = 1·4025, \quad R_{CB} = 1·3941, \quad R_{AB} = 2·7474 \text{ ohms.}$$

Hence coil in AC = 1·3533, coil in CB = 1·3449 ohms at 0° C. = 273° abs.

In liquid air at −186°·5 C.

$$R_{AC} = ·3699, \quad R_{CB} = ·3645, \quad R_{AB} = ·6937 \text{ ohms.}$$

Hence coil in AC = ·3292, coil in CB = ·3238 ohms at −186°·5 C. = 86°·5 abs.

Hence we have, using CALLENDAR's nomenclature :—

"Fundamental interval"

$$\text{for AC coil} = \frac{1·8596-1·3533}{·991} = ·5109,$$

$$\text{for CB coil} = \frac{1·8487-1·3449}{·991} = ·5084.$$

* 'Phil. Mag.,' XLVII., p. 197, 1899.

PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

"Fundamental coefficient"*

$$\text{for AC coil} = \frac{\cdot 5109}{135 \cdot 33} = \cdot 003775,$$

$$\text{for CB coil} = \frac{\cdot 5084}{134 \cdot 49} = \cdot 003780.$$

"Fundamental zero"

$$\text{for AC coil} = \frac{1}{\cdot 003775} = 264 \cdot 9 \text{ pt. degrees,}$$

$$\text{for CB coil} = \frac{1}{\cdot 003786} = 264 \cdot 6 \text{ pt. degrees.}$$

These results show that the two coils are practically identical in their properties.

Taking the means of the constants of the two coils we have temperature t_P on platinum scale corresponding to resistance $R = \frac{R - 1 \cdot 3491}{\cdot 005096}$. Hence temperature of liquid air $= \frac{\cdot 3265 - 1 \cdot 3491}{\cdot 005096} = -200 \cdot 6$ pt. degrees, which corresponds to $-186^\circ \cdot 5$ C., or $86^\circ \cdot 5$ absolute. Hence for the δ of CALLENDAR's formula

$$t - t_p = \delta \left(\frac{t}{100} - 1 \right) \frac{t}{100},$$

where t is the temperature on the hydrogen, t_p that on the platinum scale, we have

$$\delta = 14 \cdot 1 / (2 \cdot 865 \times 1 \cdot 865) = 14 \cdot 1 / 5 \cdot 35 = 2 \cdot 64.$$

Hence the platinum wire agrees closely in its properties with that used by DEWAR,† and the platinum temperature scale given by it will not differ materially from the normal platinum scale.

It was originally proposed to reduce all temperatures to the normal scale of the hydrogen thermometer, but since the δ of CALLENDAR's formula in the case of pure platinum compared with the hydrogen thermometer at 0°, 100°, and the boiling-point of sulphur, appears to have the value 1·5, and when compared at 100°, 0°, and the boiling-point of liquid oxygen to have the value 2·6, it is still somewhat uncertain what the relation between the platinum and hydrogen thermometers is at low temperatures. This fact seemed to render it advisable to give the results in terms of the platinum scale, and along with them the results of conversion to the hydrogen scale, on the assumption that CALLENDAR's δ is constant. When the connection between the two scales at low temperatures is more definitely known, the results can then be more accurately calculated in terms of the hydrogen scale.

* For pure platinum this should be ·00389.

† 'Roy. Soc. Proc.,' vol. 68, p. 363, 1901.

To carry out the conversion on the above assumption we have, since $t-t_p = \delta\left(\frac{t}{100}-1\right)\frac{t}{100}$,

$$k_{\mathrm{H}} = \frac{\mathrm{H}}{\frac{dt}{dx}} = \frac{\mathrm{H}}{\frac{dt_p}{dx}}\cdot\frac{dt_p}{dt} = k_{\mathrm{P}}\cdot\frac{dt_p}{dt} = k_{\mathrm{P}}\left\{1-\delta\left(\frac{t}{5000}-{\cdot}01\right)\right\},$$

from which the following table has been calculated :—

t_{H}.	t_{Pt}.	$\frac{k_{\mathrm{H}}}{k_{\mathrm{Pt}}}$.	t_{H}.	t_{Pt}.	$\frac{k_{\mathrm{H}}}{k_{\mathrm{Pt}}}$.
°	°		°	°	
+100	+100	·974	−100	−105·3	1·079
+ 90	+ 90·2	·979	−110	−116·1	1·084
+ 80	+ 80·4	·984	−120	−127·0	1·090
+ 70	+ 70·6	·990	−130	−137·9	1·095
+ 60	+ 60·6	·995	−140	−148·9	1·100
+ 50	+ 50·6	1·00	−150	−159·9	1·105
+ 40	+ 40·6	1·005	−160	−171·0	1·111
+ 30	+ 30·6	1·010	−170	−182·1	1·116
+ 20	+ 20·4	1·016	−180	−193·3	1·121
+ 10	+ 10·2	1·021	−190	−204·6	1·126
0	0	1·026	−200	−215·8	1·132
− 10	− 10·3	1·031	−210	−227·2	1·137
− 20	− 20·6	1·037	−220	−238·6	1·143
− 30	− 31·0	1·042	−230	−250·0	1·148
− 40	− 41·5	1·048	−240	−261·5	1·153
− 50	− 52·0	1·053			
− 60	− 62·5	1·058			
− 70	− 73·1	1·063			
− 80	− 83·8	1·069			
− 90	− 94·5	1·074			

Measurement of Temperature.

The most important of the temperature observations was that of the difference of temperature between the two platinum spirals.

Since the copper wires connecting the two spirals to the resistance bridge were of the same length, the resistance it was necessary to place in series with one of the coils when, after an adjustment to a balance against two equal resistances, the other coil was heated, was equal to the difference of the resistance of the two coils. If the coils had been of exactly the same resistance when at the same temperature, the difference of mean temperature of the coils would have been proportional to this observed difference of resistance. As they were not quite equal, a second observation was taken with the coil formerly cold now heated and the other cold. This is better than taking an observation of the difference of resistance of the two coils when at the same temperature, as it eliminates at the same time errors due to want of symmetry in the positions of the coils and heaters in the cylinder of material tested.

If t_1 and t_2 are the temperatures and R_1 and R_2 the resistances of the two coils between AC and CB respectively, in the first case ($t_1 > t_2$) we have

$$\left.\begin{aligned} t_1 &= \frac{R_1}{\cdot 005109} - 264\cdot 9 \\ t_2 &= \frac{R_2}{\cdot 005084} - 264\cdot 6 \end{aligned}\right\},$$

and $t'_1 t'_2$ in the second ($t'_2 > t'_1$) we have

$$\left.\begin{aligned} t'_1 &= \frac{R'_1}{\cdot 005109} - 264\cdot 9 \\ t'_2 &= \frac{R_2'}{\cdot 005084} - 264\cdot 6 \end{aligned}\right\}.$$

Therefore

$$R_1 - R_2 = \quad \cdot 005109 t_1 - \cdot 005084 t_2 + 264\cdot 9\,(\cdot 005109) - 264\cdot 6\,(\cdot 005084),$$

$$R'_2 - R_1 = -\cdot 005109 t'_1 + \cdot 005084 t'_2 - 264\cdot 9\,(\cdot 005109) + 264\cdot 6\,(\cdot 005084).$$

Hence

$$R_1 - R_2 + R'_2 - R_1 = \cdot 005109\,(t_1 - t'_1) + \cdot 005084\,(t'_2 - t_2),$$

or, since $t_1 - t_2 = t'_2 - t'_1$ nearly,

$$\frac{R_1 - R_2 + R'_2 - R_1}{2} = \cdot 005096\,\frac{t_1 - t_2 + t'_2 - t'_1}{2},$$

or

$$\text{Mean temperature difference between coils} = \frac{\text{Mean difference of resistance}}{\cdot 005096},$$

or

$$\text{Mean } \Delta t = 196\cdot 2 \text{ mean } \Delta R.$$

In addition to the difference of temperature of the two coils we require, if the conductivity varies with temperature, to know the mean temperature of the material between the two points at which the difference is measured.

Since the conductivity does not appear to change very rapidly with temperature, this mean temperature need not be determined with the same degree of accuracy as the temperature difference. To determine this mean temperature, the temperature of the hotter of the two coils used in measuring the difference of temperature was determined by making it one arm of a resistance bridge, of which two other arms had a constant ratio, and the fourth was adjustable. The resistances of the fixed arms were 4·850 and 8·535 ohms respectively, and the adjustable arm (r) had therefore when a balance was produced 1·762 times the resistance of the coil to be measured.

Since the two coils used in the difference measurement are nearly alike in resistance, it will be sufficient to make the calculation as if they were each equal to their mean.

The mean resistance of the coils and leads when at 0° C. was found = 1·3983, and at −200·6 Pt. = ·3672.

Taking the resistance of the leads (about ·04 ohm) to vary uniformly throughout this range, we have, if t_P is the temperature of the coil,

$$t_P = \frac{R - 1{\cdot}3983}{1{\cdot}0311} \,.\, 200{\cdot}6,$$

$$= \frac{\frac{r}{1{\cdot}762} - 1{\cdot}3983}{1{\cdot}0311} \,.\, 200{\cdot}6,$$

$$= (r - 2{\cdot}464)\, 1{\cdot}105.$$

The mean temperature of the material is obtained by subtracting from this half the observed temperature difference Δt_P, and is therefore equal to $t_P - \frac{1}{2}\Delta t_P$.

Measurement of Energy supplied.

The current (about 1 ampère) supplied to the heating-wire was measured on a Weston ammeter standardised by comparison with a Kelvin balance, whose constant had been checked by copper deposition.

The difference of potential at the ends of the leads, which consisted of 78 centims. of No. 22 copper-wire, was measured by a Weston voltmeter standardised by comparison with a Clark cell.

From the two sets of observations the resistance of each coil and its leads, for the following values of the resistance r of one of the platinum spirals, was found to be :—

$r - 2{\cdot}464$ ohms.	Platinum temperature of hot coil.	Resistance, ohms.
−1·61	−178°	·493
− ·57	− 63	·515
0	0	·528

Twenty-eight centims. of the copper leads were in air at the temperature of the room, and 50 centims. in the Dewar tube, at a mean temperature which may be taken to be 20° C. below that of the hotter platinum spiral. The resistance of the wire in the air = ·0121 ohm, that in the tube ·0216 ohm, both at 15° C., *i.e.*, when the temperature of the hotter spiral = 35° Pt. scale.

The resistance of the copper wire in the Dewar tube will become zero when its mean temperature = −237 platinum degrees on DEWAR and FLEMING'S* standard

* DEWAR and FLEMING, 'Phil. Mag.,' 36, p. 287 (1893).

thermometer, *i.e.*, when the hydrogen thermometer reads $-217°$.* Hence when the platinum thermometer used in this work reads $-235°$, *i.e.*, when the temperature indicated by the hotter spiral is -215 platinum degrees, the resistance of the copper leads in the Dewar tube will be zero.

The resistance of the coil itself at any required temperature is best determined from these data graphically, on the assumption that the change of resistance is proportional to the change of temperature (fig. 8).

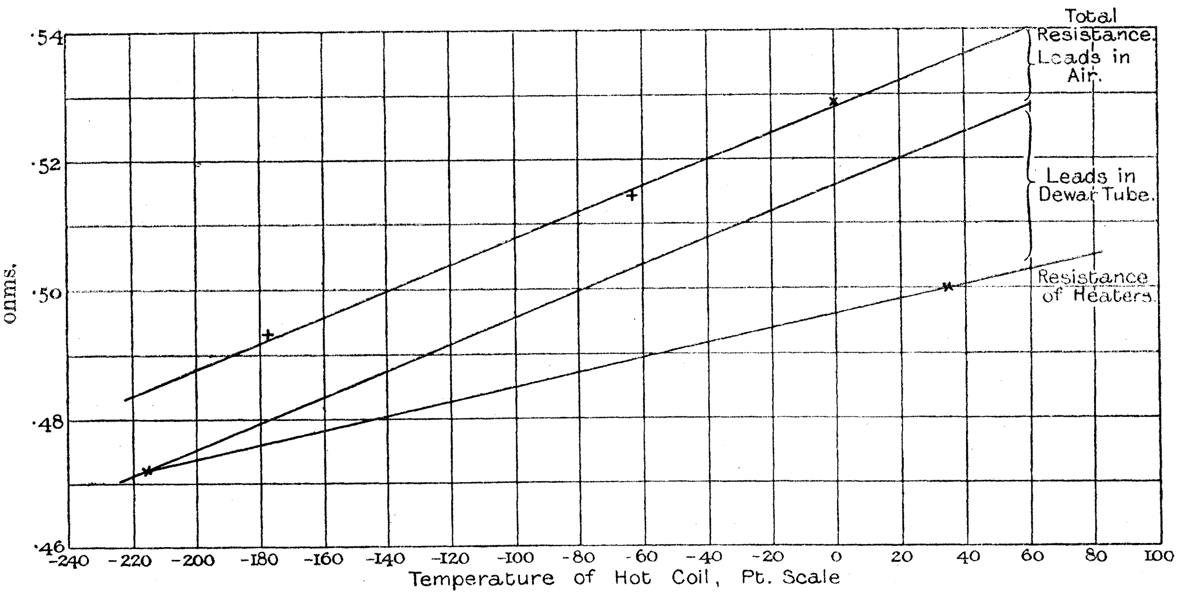

Fig. 8. Resistance of heating coils.

If C ampères are sent through the heating wire, the heat generated in it per second $= \dfrac{RC^2}{4 \cdot 2}$ gramme degrees, and since the length of the wire is 5·6 centims., the heat H generated per second per centimetre $= \dfrac{RC^2}{5 \cdot 6 \times 4 \cdot 2}$.

But we have found (p. 449) that

$$\Delta t = \bar{v}_1 - \bar{v}_2 = \frac{H}{2\pi k} 1 \cdot 46.$$

Therefore

$$k_P = \cdot 00988 \frac{RC^2}{\Delta t},$$

where R is the resistance of the heating wire, and k_P is expressed in terms of the platinum temperature scale.

In the following tables k_P is determined from the observations by means of this equation, and the corresponding value of k_H in terms of the hydrogen temperature scale calculated by the equation, p. 452, on the assumption that CALLENDAR'S δ is constant.

* CALLENDAR, 'Phil. Mag.,' 47, table, p. 214 (1899).

Paraffin Wax. (M.P. = 54° C.)

The brass tube was filled by heating paraffin wax in it over a water bath, heating the measuring apparatus in an air bath to the same temperature, then inserting it into the melted paraffin in the tube, which was then cooled slowly from underneath. As the paraffin contracted on solidifying, more was poured in at the top of the tube till the whole was solid.

CURRENT in Heaters ·991 ampère. $C^2 = \cdot 982$. $k_P = \cdot 00971 \frac{R}{\Delta t}$.

$r - 2 \cdot 46$ ohms.	t of hot coil Pt. scale.	Mean ΔR ohms.	Mean Δt Pt. scale.	R of heaters, ohms.	·00971 R.	k_P.	Mean t Pt. scale.	k_H.	Mean t H. scale.
+ ·13	+ 14°	·0434	8·51°	·498	·00484	·000569	+ 10°	·000580	+ 10° = 283° abs.
− 1·49	− 165	·0421	8·25	·478	·00464	·000563	− 169	·000625	− 158 = 115 ,,
− 1·44	− 159	·0438	8·59	·478	·00464	·000542	− 163	·000599	− 153 = 120 ,,
− 1·20	− 133	·0408	8·00	·481	·00467	·000583	− 137	·000638	− 129 = 144 ,,
− ·92	− 102	·0401	7·86	·485	·00471	·000600	− 106	·000647	− 101 = 172 ,,
− ·71	− 78	·0402	7·88	·487	·00473	·000601	− 82	·000642	− 78 = 195 ,,
− ·47	− 52	·0406	7·96	·491	·00477	·000599	− 56	·000632	− 54 = 219 ,,
− ·30	− 33	·0414	8·15	·492	·00478	·000587	− 37	·000613	− 36 = 237 ,,
− ·15	− 17	·0419	8·21	·494	·00480	·000585	− 21	·000606	− 20 = 253 ,,
− ·03	− 3	·0426	8·35	·496	·00482	·000577	− 7	·000594	− 7 = 266 ,,

With these results may be compared that obtained by the "divided bar" method (LEES, 'Phil. Trans. Roy. Soc.,' A, 183, p. 481 (1892)). $k = \cdot 00061$ between 25° C. and 45° C.

Naphthaline. (M.P. = 79° C.)

The tube was filled by heating to 100° C. in a water bath, and cooling from underneath as in the case of paraffin wax.

CURRENT in Heaters ·991 ampère. $C^2 = ·982$. $k_P = ·00971 \frac{R}{\Delta t}$.

$r - 2·46$ ohms.	t of hot coil Pt. scale.	Mean ΔR ohms.	Mean Δt Pt. scale.	R of heaters, ohms.	·00971 R.	k_{Pt}.	Mean t Pt. scale.	k_H.	Mean t H. scale.
	°		°				°		° °
+ ·56	+ 62	·0345	6·77	·503	·00488	·000721	+ 59	·000718	+ 58 = 331 abs.
−1·48	−163	·0202	3·96	·478	·00464	·00117	−165	·00130	−155 = 118 „
−1·32	−146	·0211	4·14	·480	·00466	·00113	−148	·00124	−139 = 134 „
−1·19	−132	·0220	4·32	·481	·00467	·00108	−134	·00118	−126 = 147 „
−1·01	−111	·0232	4·55	·484	·00470	·00103	−113	·00111	−107 = 166 „
−0·76	− 84	·0246	4·83	·487	·00473	·000980	− 86	·00105	− 82 = 191 „
−0·54	− 60	·0260	5·10	·489	·00475	·000931	− 63	·000985	− 60 = 213 „
−0·35	− 39	·0273	5·35	·492	·00478	·000893	− 42	·000936	− 40 = 233 „
−0·17	− 19	·0289	5·67	·494	·00480	·000847	− 22	·000879	− 21 = 252 „
−0·01	− 1	·0306	6·00	·496	·00482	·000803	− 4	·000826	− 4 = 269 „
+0·14	+ 16	·0319	6·26	·498	·00484	·000773	+ 13	·000788	+ 13 = 286 „
+0·23	+ 25	·0325	6·38	·499	·00485	·000760	+ 22	·000773	+ 22 = 295 „

With these results may be compared two obtained by placing a disc of naphthaline, ·15 centim. thick, between two copper plates and measuring the difference of temperature between the two when a known quantity of heat produced electrically was transmitted (LEES, 'Phil. Trans.,' A, vol. 191, p. 416 (1898)). k at 33° C. = ·00096, at 62° C. = ·00084.

The somewhat higher values obtained in the older experiments may be due to the fact that the method of producing the disc of naphthaline appeared to develop in it columns of crystals whose axes were perpendicular to the surfaces of the disc and therefore parallel to the lines of flow of heat.

β. Naphthol. (M.P. = 122° C.)

The tube was filled by heating in an oil bath, and cooling as described under paraffin.

CURRENT in Heaters ·991 ampère. $C^2 = \cdot 982$. $k_P = \cdot 00971 \dfrac{R}{\Delta t}$.

$r - 2\cdot 46$ ohms.	t of hot coil Pt. scale.	Mean ΔR ohms.	Mean Δt Pt. scale.	R of heaters, ohms.	·00971 R.	k_P.	Mean t Pt. scale.	k_H.	Mean t H. scale.
	°		°				°		° °
+0·87	+ 96	·0434	8·51	·507	·00492	·000578	+ 92	·000566	+ 92 = 365 abs.
−1·60	−177	·0378	7·41	·476	·00462	·000623	−181	·000695	−169 = 104 „
−1·32	−146	·0397	7·79	·480	·00466	·000598	−150	·000658	−141 = 132 „
−1·02	−113	·0396	7·77	·484	·00470	·000604	−117	·000655	−111 = 162 „
−0·74	− 82	·0403	7·90	·487	·00473	·000599	− 86	·000641	− 82 = 191 „
−0·52	− 57	·0406	7·96	·490	·00476	·000598	− 61	·000632	− 59 = 214 „
−0·31	− 34	·0406	7·96	·492	·00478	·000601	− 38	·000629	− 37 = 236 „
−0·10	− 11	·0409	8·02	·495	·00481	·000600	− 15	·000620	− 15 = 258 „
+0·44	+ 49	·0405	7·94	·502	·00487	·000613	+ 45	·000614	+ 44 = 317 „

With these results may be compared those obtained by testing a disc, ·17 centim. thick, sawn from a block of the material and placed between copper plates, as in the case of Naphthaline (LEES, 'Phil. Trans.,' A, vol. 191, p. 416 (1898)). k at 32° C. = ·00081, at 61° C. = ·00062. There seems no evidence in the present case of the higher conductivity at about 30° C. found in the former experiments.

Ice.

Obtained by filling the brass tube with air-free distilled water and slowly freezing it from below upwards by inserting the base of the tube into liquid air.

CURRENT in Heaters 1·989 ampères. $C^2 = 3·956$. $k_P = ·0391 \frac{R}{\Delta t}$.

$r - 2·46$ ohms.	t of hot coil Pt. scale.	Mean ΔR ohms.	Mean Δt Pt. scale.	R of heaters, ohms.	·0391 R.	k_P.	Mean t Pt. scale.	k_H.	Mean t H. scale.
	°		°				°		° °
−1·77	−196	·0140	2·75	·474	·0185	·00673	−197	·00755	−182 = 91 abs.
−1·70	−188	·0151	2·96	·475	·0186	·00628	−189	·00703	−176 = 97 ,,
−1·67	−185	·0158	3·10	·475	·0186	·00600	−187	·00671	−174 = 99 ,,
−1·64	−181	·0162	3·18	·476	·0186	·00585	−183	·00653	−172 = 101 ,,
−1·61	−178	·0164	3·22	·476	·0186	·00578	−180	·00645	−168 = 105 ,,
−1·60	−177	·0164	3·22	·476	·0186	·00578	−179	·00644	−167 = 106 ,,
−1·52	−168	·0168	3·30	·477	·0187	·00567	−170	·00629	−159 = 114 ,,
−1·19	−131	·0177	3·47	·482	·0188	·00542	−133	·00592	−126 = 147 ,,
−0·97	−107	·0178	3·49	·484	·0189	·00542	−109	·00586	−103 = 160 ,,
−0·89	− 98	·0181	3·55	·485	·0190	·00535	−100	·00576	− 95 = 178 ,,
−0·80	− 88	·0182	3·57	·486	·0190	·00532	− 90	·00570	− 86 = 187 ,,
−0·72	− 80	·0183	3·59	·487	·0190	·00529	− 82	·00565	− 78 = 195 ,,
−0·61	− 67	·0184	3·61	·488	·0191	·00529	− 69	·00561	− 66 = 207 ,,
−0·52	− 57	·0187	3·67	·490	·0192	·00523	− 59	·00552	− 57 = 216 ,,
−0·43	− 48	·0189	3·71	·491	·0192	·00518	− 50	·00545	− 48 = 225 ,,
−0·35	− 39	·0193	3·79	·492	·0192	·00507	− 41	·00531	− 40 = 233 ,,
−0·23	− 25	·0199	3·90	·493	·0193	·00494	− 27	·00514	− 26 = 247 ,,
−0·13	− 14	·0196	3·85	·495	·0194	·00504	− 16	·00521	− 16 = 257 ,,

With these results may be compared the following values :—

F. NEUMANN, 'Ann. Chim. Phys.,' 3, p. 66 (1862), $k = ·00568$.

MITCHELL, 'Proc. Roy. Soc. Edin.,' 86, p. 592 (1885), $k = ·005$.

It will be noticed that the conductivity of ice near its melting-point is between three and four times that of water, which H. F. WEBER has found to be ·0012 at 4° C. and several observers have found to be about ·0014 at 20° C.

Glycerine. (Pure, Sp. g. = 1·263.)

Obtained by filling the brass tube with the liquid and slowly freezing it from below upwards by dipping the base of the tube into liquid air.

CURRENT in Heaters ·991 ampère. C^2 = ·982. $k_P = \cdot 00971 \frac{R}{\Delta t}$.

$r - 2\cdot46$ ohms.	t of hot coil Pt. scale.	Mean ΔR ohms.	Mean Δt Pt. scale.	R of heaters, ohms.	·00971 R.	k_P.	Mean t Pt. scale.	k_H.	Mean t H. scale.
	° C.		° C.				° C.		° C. °
−1·64	−181	·0345	6·77	·476	·00462	·000682	−184	·000762	−172 = 101 abs.
−1·52	−168	·0340	6·67	·477	·00463	·000694	−171	·000770	−160 = 113 ,,
−1·41	−156	·0333	6·53	·479	·00465	·000712	−159	·000787	−149 = 124 ,,
−1·22	−135	·0324	6·36	·481	·00467	·000734	−138	·000804	−130 = 143 ,,
−1·08	−119	·0320	6·28	·482	·00468	·000745	−122	·000810	−115 = 158 ,,
−0·97	−107	·0316	6·20	·484	·00470	·000758	−110	·000820	−105 = 168 ,,
−0·88	− 97	·0315	6·18	·485	·00471	·000762	−100	·000821	− 95 = 178 ,,
−0·78	− 86	·0310	6·08	·486	·00472	·000776	− 89	·000831	− 85 = 188 ,,
−0·72	− 80	·0312	6·12	·487	·00473	·000773	− 83	·000826	− 79 = 194 ,,
−0·64	− 71	·0316	6·20	·488	·00474	·000765	− 74	·000813	− 71 = 202 ,,
−0·61	− 67	·0320	6·28	·489	·00475	·000756	− 70	·000802	− 67 = 206 ,,
−0·52	− 57	·0321	6·30	·490	·00476	·000755	− 60	·000798	− 58 = 215 ,,
−0·45	− 50	·0325	6·38	·491	·00477	·000748	− 53	·000788	− 51 = 222 ,,
−0·39	− 43	·0329	6·45	·491	·00477	·000740	− 46	·000777	− 44 = 229 ,,
−0·34	− 38	·0334	6·55	·492	·00478	·000730	− 41	·000765	− 40 = 233 ,,
−0·28	− 31	·0336	6·59	·493	·00479	·000727	− 34	·000758	− 33 = 240 ,,
−0·20	− 22	·0340	6·67	·494	·00480	·000720	− 25	·000748	− 24 = 249 ,,
−0·11	− 12	·0342	6·71	·495	·00481	·000717	− 15	·000741	− 15 = 258 ,,
+0·20	+ 22	·0346	6·79	·499	·00485	·000714	+ 19*	·000725	+ 19 = 292 ,,

With these results may be compared the following values for k for the liquid:—

H. F. WEBER, 'Berliner Ber.,' 1885, p. 809, ·00067, 9° C. to 15° C.

GRAETZ, 'Ann. der Phys.,' 25, p. 337 (1885), ·00062 at 13° C.

LEES, 'Phil. Trans. Roy. Soc.,' A, vol. 191, p. 424 (1898), ·00068 at 25° C., ·00061 at 48° C.

* Probably liquid, as the melting-point of glycerine is given by RICHTER as 17° C.

Aniline. (Re-distilled, B.P. 183–183·5° C.; slight orange colour.)

Obtained by freezing from below as in previous cases.
The freezing-point is given by RICHTER as −8° C.

CURRENT in Heaters ·991 ampère. $C^2 = ·982$. $k_P = ·00971 \frac{R}{\Delta t}$.

$r - 2·46$ ohms.	t of hot coil Pt. scale.	Mean ΔR ohms.	Mean Δt Pt. scale.	R of heaters, ohms.	·00971 R.	k_P.	Mean t Pt. scale.	k_H.	Mean t H. scale.
	°		°				°		° °
−1·74	−192	·0195	3·83	·475	·00461	·00120	−194	·00135	−181 = 92 abs.
−1·51	−167	·0921*	18·07	·478	·01869	·00103	−176	·00115	−165 = 108 „
−1·30	−144	·0254	4·98	·480	·00466	·000936	−146	·00103	−137 = 136 „
−1·10	−122	·0272	5·34	·483	·00469	·000878	−125	·000956	−118 = 155 „
−1·00	−110	·0289	5·67	·484	·00470	·000829	−113	·000898	−107 = 166 „
−0·89	− 98	·0300	5·89	·485	·00471	·000800	−101	·000861	− 96 = 177 „
−0·79	− 87	·0310	6·08	·486	·00472	·000776	− 90	·000832	− 86 = 187 „
−0·68	− 75	·0324	6·36	·488	·00474	·000745	− 78	·000792	− 75 = 198 „
−0·58	− 64	·0337	6·61	·489	·00475	·000710	− 67	·000762	− 64 = 209 „
−0·48	− 53	·0349	6·85	·490	·00476	·000695	− 56	·000733	− 54 = 219 „
−0·39	− 43	·0360	7·06	·491	·00477	·000676	− 47	·000710	− 45 = 228 „
−0·36	− 40	·0369	7·24	·492	·00478	·000660	− 44	·000692	− 42 = 231 „
−0·33	− 36	·0363	7·12	·492	·00478	·000671	− 40	·000702	− 39 = 234 „

With these results may be compared H. F. WEBER's result for the liquid, 'Berliner Ber.,' 1885, p. 809. $k = ·000408$ between 9° C. and 15° C.

* Current in heaters 1·989 ampère. $C^2 = 3·956$. $k_P = ·0391 \frac{R}{\Delta t}$.

Diphenylamine. (From KAHLBAUM, M.P. = 54° C.)

Obtained by melting the crystals over a water bath, filling the brass tube, and freezing slowly from beneath.

CURRENT in Heaters ·991 ampère. $C^2 = \cdot 982$. $k_P = \cdot 00971 \dfrac{R}{\Delta t}$.

$r - 2\cdot46$ ohms.	t of hot coil Pt. scale.	Mean ΔR ohms.	Mean Δt Pt. scale.	R of heaters, ohms.	·00971 R.	k_P.	Mean t Pt. scale.	k_H.	Mean t H. scale
	°		°				°		° °
+0·28	+ 31	·0489	9·59	·500	·00486	·000507	+ 26	·000517	+ 26 = 299 abs.
−1·73	−191	·0438	8·59	·475	·00461	·000537	−195	·000603	−182 = 91 ,,
−1·42	−157	·0456	8·94	·479	·00465	·000520	−161	·000575	−151 = 122 ,,
−1·21	−134	·0462	9·06	·481	·00467	·000515	−139	·000564	−131 = 142 ,,
−1·10	−122	·0462	9·06	·483	·00469	·000518	−127	·000565	−120 = 153 ,,
−1·00	−110	·0474	9·30	·484	·00470	·000505	−115	·000547	−109 = 164 ,,
−0·91	−101	·0476	9·34	·485	·00471	·000504	−106	·000544	−101 = 172 ,,
−0·82	− 91	·0476	9·34	·486	·00472	·000505	− 96	·000543	− 91 = 182 ,,
−0·73	− 81	·0475	9·32	·487	·00473	·000508	− 86	·000544	− 82 = 191 ,,
−0·64	− 71	·0474	9·30	·488	·00474	·000510	− 76	·000543	− 73 = 200 ,,
−0·60	− 66	·0472	9·26	·489	·00475	·000513	− 71	·000545	− 68 = 205 ,,
−0·51	− 56	·0475	9·32	·490	·00476	·000511	− 61	·000540	− 59 = 214 ,,
−0·48	− 53	·0480	9·42	·490	·00476	·000505	− 58	·000533	− 56 = 217 ,,
−0·41	− 45	·0484	9·50	·491	·00477	·000502	− 50	·000528	− 48 = 225 ,,
−0·36	− 40	·0484	9·50	·492	·00478	·000503	− 45	·000528	− 43 = 230 ,,
−0·31	− 34	·0485	9·52	·492	·00478	·000502	− 39	·000525	− 38 = 235 ,,
−0·25	− 28	·0485	9·52	·493	·00479	·000503	− 33	·000524	− 32 = 241 ,,
−0·20	− 22	·0481	9·44	·494	·00480	·000509	− 27	·000529	− 26 = 247 ,,
−0·16	− 18	·0479	9·40	·494	·00480	·000511	− 23	·000530	− 22 = 251 ,,
−0·11	− 12	·0476	9·34	·495	·00481	·000515	− 17	·000532	− 17 = 256 ,,
−0·02	− 2	·0483	9·48	·496	·00482	·000508	− 7	·000523	− 7 = 266 ,,
+0·07	+ 7	·0485	9·52	·497	·00483	·000507	+ 2	·000521	+ 2 = 275 ,,

Pure Nitrophenol. (From Kahlbaum, M.P. = 114° C.)

Cylinder prepared as described under Naphthol.

Current in Heaters ·991 ampère. C^2 = ·982. $k_P = {\cdot}00971 \dfrac{R}{\Delta t}$.

$r - 2{\cdot}46$ ohms.	t of hot coil Pt. scale.	Mean ΔR ohms.	Mean Δt Pt. scale.	R of heaters, ohms.	·00971 R.	k_P.	Mean t Pt. scale.	k_H.	Mean t H. scale.
	°		°				°		° °
+0·46	+ 51	·0414	8·12	·502	·00487	·000600	+ 47	·000601	+ 46 = 319 abs.
−1·71	−189	·0234	4·59	·475	·00461	·00100	−191	·00112	−178 = 95 „
−1·52	−168	·0246	4·83	·477	·00463	·000959	−170	·00106	−159 = 114 „
−1·38	−152	·0263	5·16	·479	·00465	·000901	−155	·000994	−146 = 127 „
−1·26	−139	·0274	5·38	·481	·00467	·000868	−142	·000952	−134 = 139 „
−1·15	−127	·0281	5·51	·482	·00468	·000850	−130	·000927	−123 = 150 „
−1·06	−117	·0288	5·65	·483	·00469	·000830	−120	·000901	−114 = 159 „
−0·99	−109	·0296	5·81	·484	·00470	·000809	−112	·000875	−106 = 167 „
−0·89	− 98	·0305	5·98	·485	·00471	·000788	−101	·000849	− 96 = 177 „
−0·84	− 93	·0316	6·20	·486	·00472	·000761	− 96	·000818	− 91 = 182 „
−0·69	− 76	·0324	6·36	·488	·00474	·000745	− 79	·000794	− 76 = 197 „
−0·63	− 70	·0331	6·49	·488	·00474	·000730	− 73	·000776	− 70 = 203 „
−0·52	− 57	·0341	6·69	·490	·00476	·000712	− 60	·000752	− 58 = 215 „
−0·49	− 54	·0346	6·79	·490	·00476	·000701	− 57	·000740	− 55 = 218 „
−0·40	− 44	·0354	6·94	·491	·00477	·000687	− 47	·000722	− 45 = 228 „
−0·34	− 38	·0359	7·04	·492	·00478	·000679	− 42	·000712	− 40 = 233 „
−0·27	− 30	·0365	7·16	·493	·00479	·000669	− 34	·000698	− 33 = 240 „
−0·20	− 22	·0368	7·22	·494	·00480	·000665	− 26	·000692	− 25 = 248 „
−0·11	− 12	·0371	7·28	·495	·00481	·000661	− 16	·000683	− 16 = 257 „
−0·04	− 4	·0378	7·41	·496	·00482	·000650	− 8	·000670	− 8 = 265 „
0·00	0	·0383	7·51	·496	·00482	·000642	− 4	·000660	− 4 = 269 „
+0·04	+ 4	·0387	7·59	·497	·00483	·000636	0	·000653	0 = 273 „
+0·09	+ 10	·0391	7·67	·497	·00483	·000630	+ 6	·000644	+ 6 = 279 „
+0·10	+ 11	·0391	7·67	·497	·00483	·000630	+ 7	·000643	+ 7 = 280 „
+0·49	+ 54	·0416	8·16	·502	·00487	·000597	+ 50	·000597	+ 49 = 322 „

Glycerine. (Pure, Sp. g. 1·263.) (Second Experiment.)

Obtained as in first experiment.

The measurements on glycerine were repeated, as it alone of the substances tested showed a distinct maximum of conductivity within the range of the experiments. The close agreement of these measurements with those previously made gives an idea of the accuracy to be expected from the apparatus.

CURRENT in Heaters ·991 ampère. $C^2 = \cdot 982$. $k_P = \cdot 00971 \frac{R}{\Delta t}$.

$r - 2\cdot46$ ohms.	t of hot coil Pt. scale.	Mean ΔR ohms.	Mean Δt Pt. scale.	R of heaters, ohms.	·00971 R.	k_P.	Mean t Pt. scale.	k_H.	Mean t H. scale.
	° C.		° C.				° C.		° C. °
−1·64	−181	·0351	6·89	·476	·00462	·000671	−184	·000749	−172 = 101 abs.
−1·45	−160	·0336	6·59	·478	·00464	·000704	−163	·000778	−153 = 120 ,,
−1·36	−150	·0333	6·53	·479	·00465	·000712	−153	·000784	−144 = 129 ,,
−1·28	−141	·0329	6·45	·480	·00466	·000722	−144	·000793	−136 = 137 ,,
−1·21	−134	·0327	6·42	·481	·00467	·000727	−137	·000796	−129 = 144 ,,
−1·10	−122	·0326	6·40	·483	·00469	·000733	−125	·000798	−118 = 155 ,,
−1·02	−113	·0322	6·32	·483	·00469	·000742	−116	·000804	−110 = 163 ,,
−0·93	−103	·0320	6·28	·484	·00470	·000748	−106	·000807	−101 = 172 ,,
−0·89	− 98	·0316	6·20	·485	·00471	·000758	−101	·000817	− 96 = 177 ,,
−0·85	− 94	·0312	6·12	·486	·00472	·000771	− 97	·000829	− 92 = 181 ,,
−0·81	− 90	·0310	6·08	·486	·00472	·000776	− 93	·000833	− 89 = 184 ,,
−0·64	− 71	·0311	6·10	·488	·00474	·000777	− 74	·000826	− 71 = 202 ,,
−0·60	− 66	·0315	6·18	·489	·00475	·000769	− 69	·000816	− 66 = 207 ,,
−0·58	− 64	·0317	6·22	·489	·00475	·000764	− 67	·000810	− 64 = 209 ,,
−0·51	− 56	·0321	6·30	·490	·00476	·000754	− 59	·000796	− 57 = 216 ,,
−0·49	− 54	·0325	6·38	·490	·00476	·000746	− 57	·000787	− 55 = 218 ,,
−0·42	− 46	·0331	6·49	·491	·00477	·000735	− 49	·000773	− 47 = 226 ,,
−0·40	− 44	·0334	6·55	·491	·00477	·000728	− 47	·000765	− 45 = 228 ,,
−0·34	− 38	·0334	6·55	·492	·00478	·000730	− 41	·000765	− 40 = 233 ,,
−0·32	− 35	·0336	6·59	·492	·00478	·000725	− 38	·000758	− 37 = 236 ,,
−0·29	− 32	·0339	6·65	·493	·00479	·000720	− 35	·000752	− 36 = 237 ,,
−0·27	− 30	·0342	6·71	·493	·00479	·000714	− 33	·000745	− 32 = 241 ,,
−0·21	− 23	·0344	6·75	·494	·00480	·000711	− 26	·000739	− 25 = 248 ,,
−0·19	− 21	·0343	6·73	·494	·00480	·000713	− 24	·000741	− 23 = 250 ,,
−0·17	− 19	·0344	6·75	·494	·00480	·000711	− 22	·000738	− 21 = 252 ,,
−0·13	− 14	·0345	6·77	·495	·00481	·000710	− 17	·000735	− 17 = 256 ,,

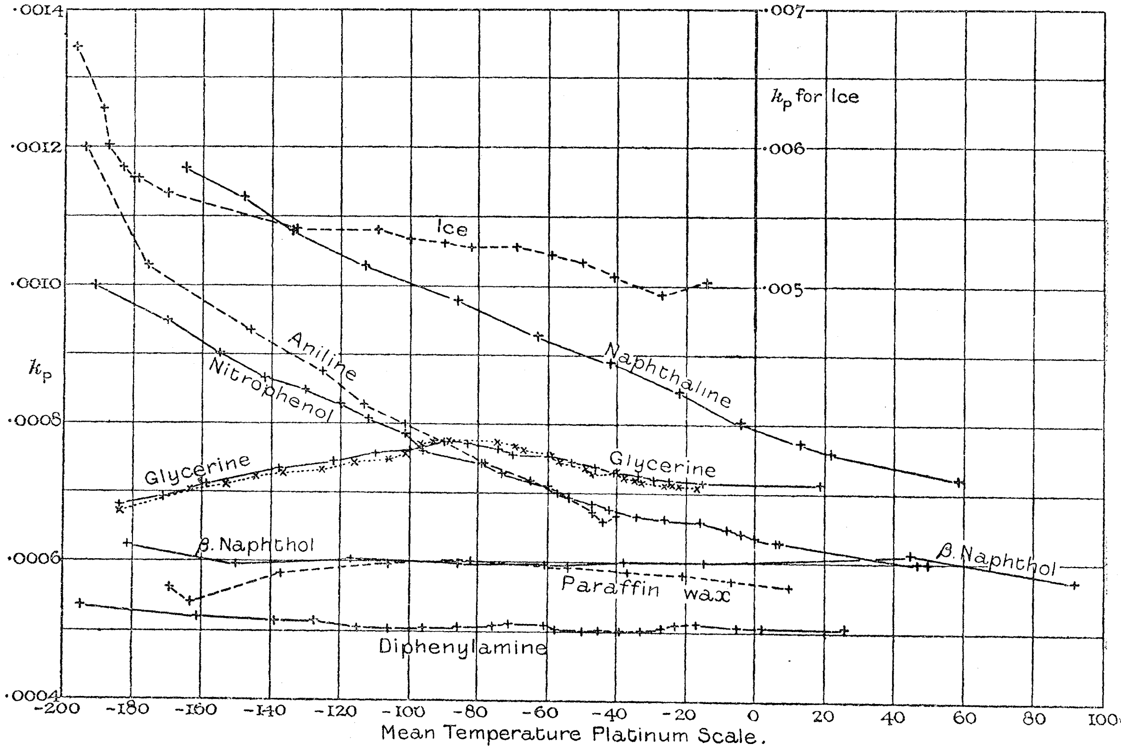

Fig. 9. Thermal conductivities in terms of Pt. temperature scale.

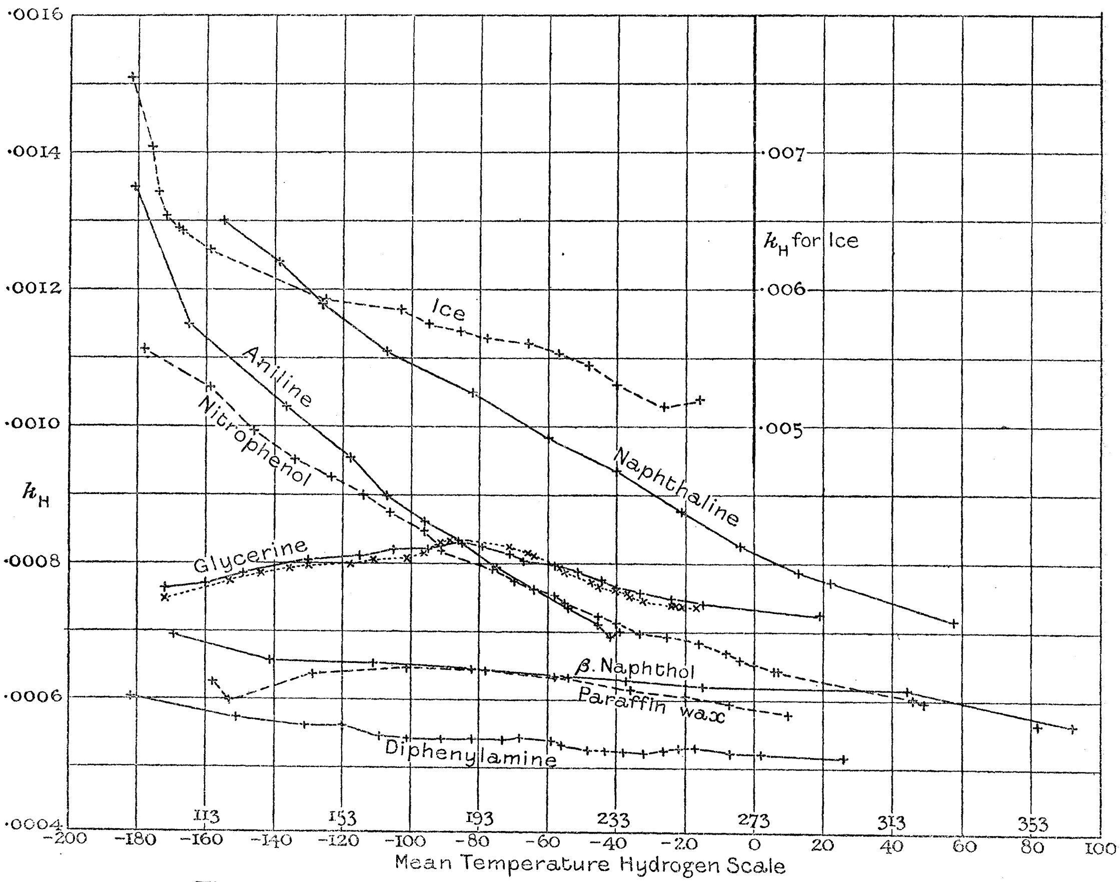

Fig. 10. Thermal conductivities in terms of H. temperature scale.

Remarks on the Preceding Results.

The agreement of the results obtained at the higher temperatures with results previously known has been shown in the notes to the tables to be very good in the cases of paraffin wax, ice, and glycerine, but not all that could be desired in the cases of naphthaline and β naphthol. It should, however, not be overlooked that crystals in general possess different conductivities in different directions, and that considerable differences may be caused by the different methods in which the crystals in a cooling mass, *e.g.*, of naphthaline, arrange themselves with respect to those directions which are ultimately to become the lines of flow of heat.

Whatever the arrangement of the crystals in any particular sample of a substance, if that sample is tested at different temperatures, the results express the variation with temperature of the thermal conductivity of that sample in some fixed direction, and enable the question as to whether conductivity increases or decreases with increase of temperature to be answered.

The curves expressing the results of the experiments show that there is a marked increase of the thermal conductivities of ice, naphthaline, aniline, and nitrophenol as the temperature is decreased, a slight increase (in terms of the hydrogen scale only) in the cases of β naphthol and diphenylamine, and possibly of paraffin wax, and an increase to a maximum at $-80°$ with a decrease beyond in the case of glycerine.

These facts point to the conclusion that the effect of temperature on the thermal conductivities of electrical insulators is mainly, if not entirely, determined by the physical and chemical nature of each substance, and cannot be stated for electrical insulators generally, although there seems on the average to be a tendency towards higher conductivities at lower temperatures.

In two cases, *i.e.*, those of ice and aniline, the thermal conductivity of the solid is much greater than that of the liquid; while in another, glycerine, the conductivities in the two states near the melting-point are almost identical.

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